Math 20550 - Summer 2016

Line Integrals, Curl, and Divergence

July 8, 2016

Problem 1.

(a) *Compute* $\int_0^2 e^x dx$.

(b) *Let* C *be the line segment from* $(0,0)$ *to* $(2,0)$ *and compute* $\int_C e^{x+y} ds$.

(c) *Let* C *be as in part (b) and compute* $\int_C e^{x+y} dx$.

(d) *Let* $-C$ *denote the line segment from* $(2,0)$ *to* $(0,0)$. *Compute* $\int_{-C} e^{x+y} ds$.

(e) *Compute* $\int_{-C} e^{x+y} dx$.

(f) *The integral in part (a) is one like you studied in Calc I. Which of the integrals* $\int_C e^{x+y} ds$ *or* $\int_C e^{x+y} dx$ *does is correspond to? Why one and not the other?*

Problem 2. *Compute* $\int_C xy\ ds$ *where* C *is parametrized by* $\mathbf{r}(t) = 4t\mathbf{i} + 3t\mathbf{j}$, $0 \le t \le 1$.

Problem 3. *Compute* $\int_C (x^2 + y^2 + z^2)\ ds$ *where* C *is parametrized by* $\mathbf{r}(t) = \sin t\mathbf{i} + \cos t\mathbf{j} + 2\mathbf{k}$, $0 \le t \le \frac{\pi}{2}$.

Problem 4. *Find the total mass of a wire bent in the shape of* $y = \sqrt{9 - x^2}$ *with linear density function* $\rho(x,y) = x + y + 2$.

Problem 5. *Compute* $\int_C \mathbf{F} \cdot d\mathbf{r}$ *where* $\mathbf{F} = \langle x, y\rangle$ *and* C *is the path which goes once around the triangle with vertices* $(0,0)$, $(1,0)$, *and* $(0,1)$ *in the counterclockwise direction.*

Problem 6. *Find the work done by the force field* $\mathbf{F} = \langle x^2, -xy\rangle$ *in moving a particle from* $(1,1)$ *to* $(9,243)$ *along the path* $x = t^2$, $y = t^5$.

Problem 7. *Consider the cylinder of radius* 2 *whose base is the disk* $x^2 + y^2 \le 4$ *in the xy-plane. Suppose we cut off the top of the cylinder according to the height function* $h(x,y) = x^2 - y^2 + 6$. *Find the lateral surface area of this cylinder (i.e., the surface area of the side, but not the base or top).*

Problem 8. *Compute* $\int_C x\ dx + y\ dy + xy\ dz$ *where* C *is the path* $\mathbf{r}(t) = \langle \cos t, \sin t, t\rangle$, $0 \le t \le \pi$.

Problem 9. *Write down* 3 *different paths from* $(0,0)$ *to* $(1,1)$. *Let* $f(x,y) = x^5y^6$ *and compute* $\int_C \nabla f \cdot d\mathbf{r}$ *for all* 3 *of your paths. What do you notice?*

Problem 10. *Find* div $\mathbf{F}$ *and* curl $\mathbf{F}$ *where* $\mathbf{F} = \langle xy + e^z, z\cos x, \ln(xyz)\rangle$.